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estimate $\frac{1}{203}$ by linear approximation

Underlying Theme: Do what the instructions say

Linear Approximation = tangent line

an approximation for $y=f(x)$ based on the tangent line at (x_0, y_0)

$$\Rightarrow \quad y = y_0 + f'(x_0)(x - x_0)$$

$$y = f(x_0) + f'(x_0)(x - x_0)$$

Need:
x_0
y_0
$f'(x_0)$

$y = \frac{1}{x}$

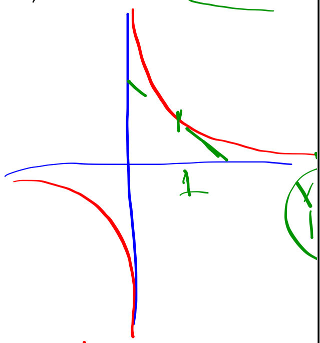
$y' = \frac{-1}{x^2}$

$x_0 = 1$

$f'(x_0) = \frac{-1}{1^2} = -1$

$y = 1 + (-1)(x - 1)$

203



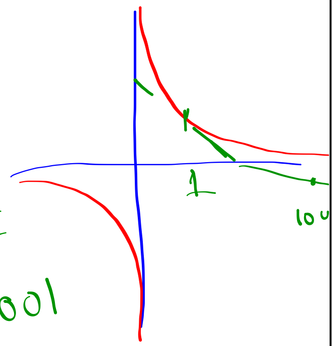
$f(203) = \frac{1}{203} \approx y = 1 - (203) + 1 = -201$

$x_0 = 100$

$y_0 = f(x_0) = \frac{1}{100} = .01$

$y'(x_0) = f'(100) = \left.\frac{-1}{x^2}\right|_{x=100} = \frac{-1}{100^2}$

$= -.0001$



$y = .01 + (-.0001)(203 - 100)$

$= .01 - .0001(103) = .01 - .0103 = -.00$

$\frac{1}{203} \approx -.00$

$x_0 = 200$

$\frac{1}{200} = .005$

$f'(x_0) = \frac{-1}{(200)^2} = \frac{-1}{40000} = -.000025$

$\frac{1}{203} \approx y = .005 + (-.000025)(203 - 200)$

Dec 5-7:25 AM

Estimate $\sqrt{17}$

day 5.13

$f(x) = \sqrt{x}$

$x_0 = 16$

$f(16) = 4 = y_0$

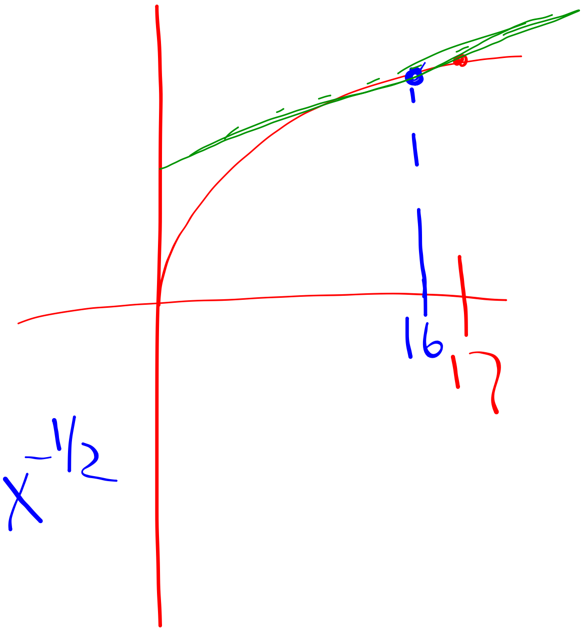
$f'(x) = \frac{d}{dx}(x^{1/2}) = \frac{1}{2}x^{-1/2}$

$= \frac{1}{2\sqrt{x}}$

$f'(16) = \frac{1}{2\sqrt{16}} = \frac{1}{8} \approx .0125$

$\sqrt{17} \approx f(16) + f'(16)(x-16) \Big|_{x=17}$

$= 4 + \frac{1}{8}(17-16) = 4 + \frac{1}{8} \approx 4.125$



day 5.13

$4 \cdot 4 = 16$

$(4.1)(4.1) = 16.81$

$(4.2)(4.2) = 17.64$

$(4.125)(4.125) = 17.015625$

16.81

$(4.120)(4.120)$

$(4.122)(4.122) =$

$$\begin{array}{r} 41 \\ \times\, 41 \\ \hline 41 \\ 164 \\ \hline 1681 \end{array} \qquad \begin{array}{r} 42 \\ \times\, 42 \\ \hline 84 \\ 168 \\ \hline 1764 \end{array}$$

$$\begin{array}{r} 4125 \\ \times\, 4125 \\ \hline 20625 \\ 8250 \\ 4125 \\ 16500 \\ \hline 17.015625 \end{array}$$

$$\begin{array}{r} 412 \\ \times\, 412 \\ \hline 824 \\ 412 \\ 1648 \\ \hline 16.9744 \end{array}$$

4.7 L'Hospital's Rule

day 5.13

L'H...

$\frac{0}{0}$

$\frac{\infty}{\infty}$

The indeterminate form of $0 \cdot \infty$

$$\lim_{x \to 0} (\sin x)\left(\frac{1}{x}\right)$$

"turn 0 into $\frac{1}{\infty}$"

OR

"turn ∞ into $\frac{1}{0}$"

$$= \lim_{x \to 0} \frac{\left(\frac{1}{x}\right)}{\left(\frac{1}{\sin x}\right)} = \lim_{x \to 0} \frac{\frac{1}{x}}{\csc x}$$

$$\overset{LH}{=} \lim_{x \to 0} \frac{-\frac{1}{x^2}}{-\csc x \cot x}$$

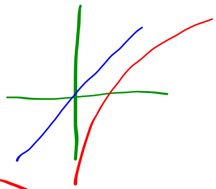
$$\lim_{x \to 0} (\sin x)\left(\frac{1}{x}\right) = \lim_{x \to 0} \frac{(\sin x)}{\frac{1}{\left(\frac{1}{x}\right)}} = \lim_{x \to 0} \frac{\sin x}{x}$$

$$\overset{LH}{=} \lim_{x \to 0} \frac{\cos x}{1} = 1$$

Dec 5-7:31 AM

day 5.13

$$\lim_{x\to 0} (\ln|x|)(x) \quad \text{"}-\infty\text{"} \quad \text{"}0\text{"}$$



$$= \lim_{x\to 0} \frac{x}{\frac{1}{\ln|x|}} \quad \text{"}\frac{0}{0}\text{"}$$

$$\lim_{x\to 0} \frac{(\ln|x|)}{\left(\frac{1}{x}\right)} \quad \text{"}\frac{\infty}{\infty}\text{"}$$

$$= \lim_{x\to 0} \frac{\frac{1}{x}}{-\frac{1}{x^2}} =$$

$$= \lim_{x\to 0} \left(\frac{1}{x}\right)\left(-x^2\right) = \lim_{x\to 0}(-x)$$

$$= 0$$

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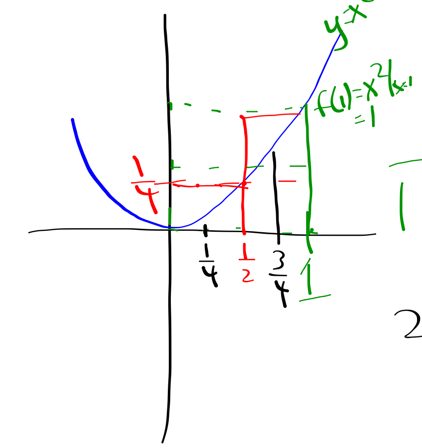
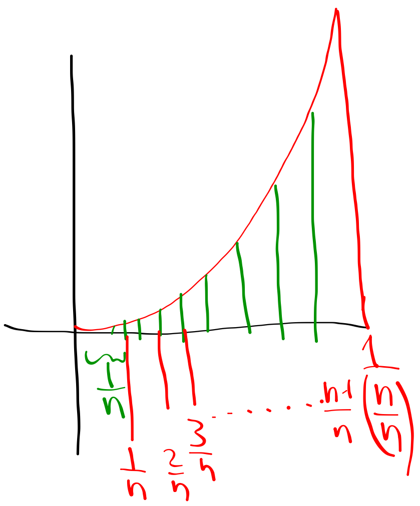
5.2) Approximating Area

day 5.13



n	RHS base·ht	LHS base·ht	MPS base·ht
1 rectangle	$1\cdot 1=1$	$1\cdot 0=0$	$1\cdot\frac{1}{4} = \frac{1}{4}$
2 rect	$\frac{1}{2}\cdot\frac{1}{4}+\frac{1}{2}\cdot 1 = \frac{5}{8}$	$\frac{1}{2}\cdot 0+\frac{1}{2}\left(\frac{1}{4}\right) = \frac{1}{8}$	$\frac{1}{2}\cdot\left(\frac{1}{4}\right)^2 + \frac{1}{2}\cdot\left(\frac{3}{4}\right)^2 = \frac{1}{32}+\frac{9}{32} = \frac{5}{16}$



n rectangles — RHS

$$\frac{1}{n}\left(f\left(\tfrac{1}{n}\right)\right)+\frac{1}{n}\left(f\left(\tfrac{2}{n}\right)\right)+\frac{1}{n}(\cdots)+\frac{1}{n}f\left(\tfrac{n}{n}\right)$$

$$=\frac{1}{n}\left(\frac{1}{n}\right)^2+\frac{1}{n}\left(\frac{2}{n}\right)^2+\frac{1}{n}\left(\frac{3}{n}\right)^2+\cdots+\frac{1}{n}\left(\frac{n}{n}\right)^2$$

$$=\sum_{k=1}^{n}\frac{1}{n}\left(\frac{k}{n}\right)^2$$

$$\sum_{k=1}^{n}\left(\frac{1}{n}\right)\frac{k^2}{n^2}=\sum_{k=1}^{n}\frac{k^2}{n^3}$$

$$=\frac{1}{n^3}\sum_{k=1}^{n}k^2$$

$$=\frac{1}{n^3}\left(\frac{n\cdot(n+1)(2n+1)}{6}\right)$$

For n rect.

$$\text{area}=\frac{n(n+1)(2n+1)}{6n^3}$$

$$\text{Area}=\lim_{n\to\infty}\frac{n(n+1)(2n+1)}{6n^3}=\frac{1}{3}$$

day 5.13

1, 4, 9, 16, 25, 36, ...

1 2

5, 14, 30, 55, 91, ...

$\frac{1}{3}$ $\frac{4}{3}$ $\frac{9}{3}$

$\frac{14}{3}$ $\frac{38}{3}$ - $\frac{81}{3}$ $\frac{149}{3}$ ...

D2 , , 11, ...

2·1

D3 9 2 2 2

3·2·1

$\text{Patt} = \frac{2}{6}n^3$

D1 , 16, 25, 36 ...